ASSIGNMENT PROBLEM

Assignment Problem

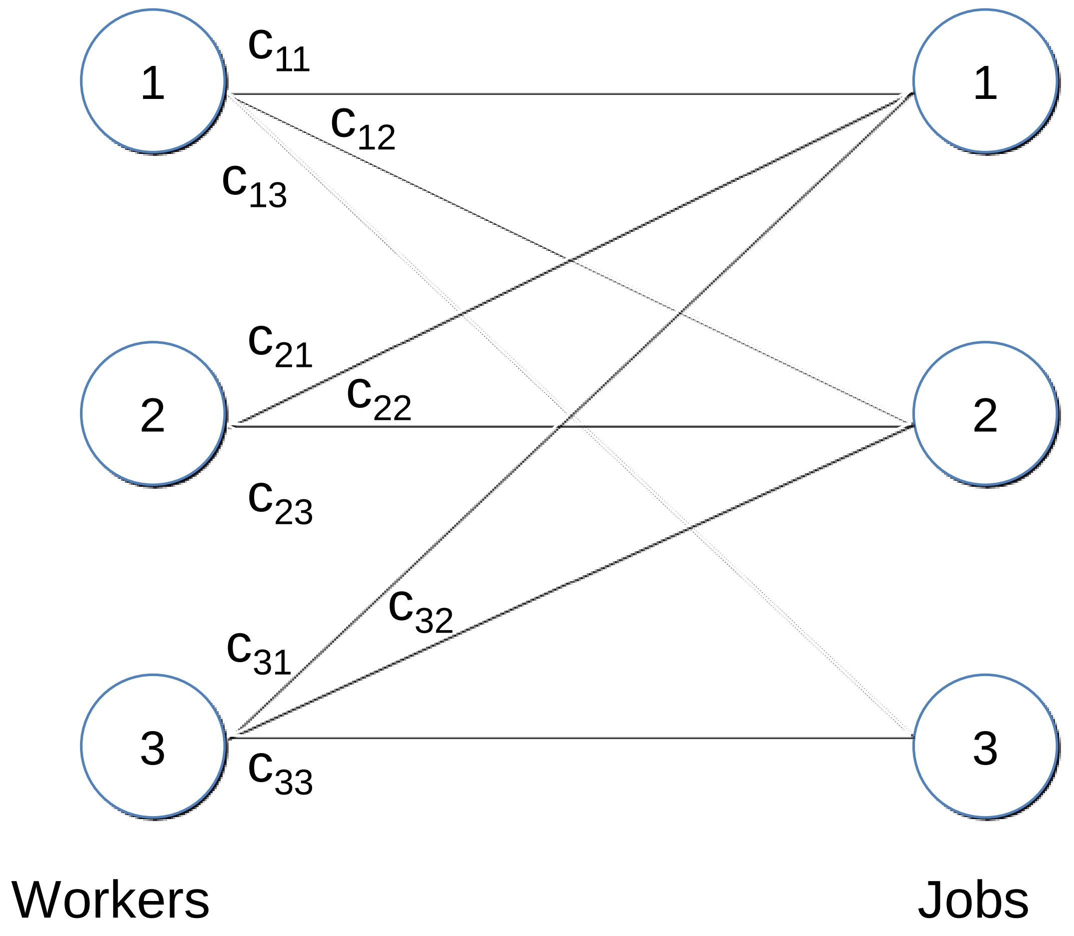
- An assignment problem seeks to minimize the total cost of assignment of *m* worker to *m* jobs, given that the cost of worker *i* performing job *j* is c_{ij}.

- It assumes all workers are assigned and each job is performed.

- An assignment problem is a special case of a transportation problem in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.

Assignment Problem

Network Representation



Assignment Problem

Linear Programming Formulation

$$\text{Min} \quad \sum_{i}\sum_{j} c_{ij}x_{ij}$$

$$\text{s.t.} \quad \sum_{j} x_{ij} = 1 \qquad \text{for each worker } i$$

$$\sum_{i} x_{ij} = 1 \qquad \text{for each job } j$$

$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i \text{ and } j$$

Assignment Problem Example

	J_1	J_2	J_3	Supply
M_1	4	8	10	1
M_2	6	9	7	1
M_3	11	12	5	1
Demand	1	1	1	3

Then

Minimise $Z = 4x_{11} + 8x_{12} + 10x_{13} + 6x_{21} + 9x_{22} + 7x_{23} + 11x_{31} + 12x_{32} + 5x_{33}$

Subject to following constraints

One man

one job $x_{11} + x_{12} + x_{13} = 1$ for man M_1

$x_{21} + x_{22} + x_{23} = 1$ for man M_2

$x_{31} + x_{32} + x_{33} = 1$ for man M_3

$\sum_{j=1}^{3} X_{ij} = 1$ for $i = 1, 2, 3.....$

The Hungarian Method

1. Subtract row minimum from each element in the row
2. Subtract column minimum from each element in the column
3. Cover the zeroes with as few lines as possible
4. If the number of lines = m = n, then optimal solution is hidden in zeroes
5. Otherwise, find the minimum cost that is not covered by any lines
    1. Subtract it from all uncovered elements
    2. Add it to all elements at intersections (covered by two lines)
6. Back to step 3

The Hungarian Method – Optimal Solution

How to identify the optimal solution:

- Make the assignments one at a time in positions that have zero elements.
- Begin with rows or columns that have only one zero. Cross out both the row and the column involved after each assignment is made.
- Move on to the rows and columns that are not yet crossed out to select the next assignment, with preference given to any such row or column that has only one zero that is not crossed out.
- Continue until every row and every column has exactly one assignment and so has been crossed out.

Solved example:

	A	B	C	D	E
1	10	4	6	10	12
2	11	7	7	9	14
3	13	8	12	14	15
4	14	16	13	17	17
5	17	11	17	20	19

Row Subtraction

	A	B	C	D	E
1	6	0	2	6	8
2	4	0	0	2	7
3	5	0	4	6	7
4	1	3	0	4	4
5	6	0	6	9	8

Column Subtraction

	A	B	C	D	E
1	5	$\boxed{0}$	2	4	4
2	3	0	$\boxed{0}$	0	3
3	4	0	4	4	3
4	$\boxed{0}$	3	0	2	0
5	5	0	6	7	4

Minimum uncovered number

	A	B	C	D	E
1	3	0	$\boxed{0}$	2	2
2	3	2	0	$\boxed{0}$	3
3	2	$\boxed{0}$	2	2	1
4	$\boxed{0}$	5	0	2	0
5	3	0	4	5	2

Minimum uncovered number

Optimal Solution

	A	B	C	D	E
1	3	1	$\boxed{0}$	2	2
2	3	3	0	$\boxed{0}$	3
3	1	0	1	1	$\boxed{0}$
4	$\boxed{0}$	6	0	2	0
5	2	$\boxed{0}$	3	4	1

JOB	TIME
1-C	**6**
2-D	**9**
3-E	**15**
4-A	**14**
5-B	**11**
	55

Example-1

There are 3 jobs *A*, *B*, and *C* and three machines *X*, *Y*, and *Z*. All the jobs can be processed on all machines. The time required for processing job on a machine is given below in the form of matrix. Make allocation to minimize the total processing time.

	Machines (time in hours)			
		X	Y	Z
Jobs	A	11	16	21
	B	20	13	17
	C	13	15	12

Example-2

A company has four jobs to be completed. Each machine must be assigned to complete one job. The time required to setup each machine for completing each job is shown in the table below. Company wants to minimize the total setup time needed to complete the four jobs.

Setup times

(Also called the cost matrix)

	Time (Hours)			
	Job1	Job2	Job3	Job4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Example-3
Multiple optimal solution

	Job (Hours)			
	A	B	C	D
Machine 1	5	3	2	8
Machine 2	7	9	2	6
Machine 3	6	4	5	7
Machine 4	5	7	7	8

Example-4
Unbalanced Problem

	Worker		
	I	II	III
Job A	9	12	11
Job B	8	13	17
Job C	20	12	13
Job D	21	15	17

Example-5
Unbalanced Problem

	Job (Hours)			
	A	B	C	D
M1	9	14	19	15
M2	7	17	20	19
M3	9	18	21	18
M4	10	12	18	19
M5	10	15	21	16

Example-6

- The ABC company has a taxi waiting at each of the four cab stands. Four customers have called and requested service. The distance in miles from the waiting taxis to the customer are given in the matrix below. Find the optimal assignment of taxis to customers so as to minimize the total driving distances to the customer.

Cost Matrix

Cab Sites	Customer (Dist in miles)			
	A	B	C	D
Stand 1	40	50	60	65
Stand 2	30	38	46	48
Stand 3	25	33	41	43
Stand 4	39	45	51	59